

Exercise 15

Find dy/dx by implicit differentiation.

$$e^{x/y} = x - y$$

Solution

Differentiate both sides with respect to x .

$$\begin{aligned}\frac{d}{dx}(e^{x/y}) &= \frac{d}{dx}(x - y) \\ (e^{x/y}) \cdot \frac{d}{dx}\left(\frac{x}{y}\right) &= \frac{d}{dx}(x) - \frac{d}{dx}(y) \\ (e^{x/y}) \cdot \frac{[\frac{d}{dx}(x)]y - [\frac{d}{dx}(y)]x}{y^2} &= (1) - (y') \\ (e^{x/y}) \cdot \frac{(1)y - (y')x}{y^2} &= 1 - y' \\ e^{x/y} \cdot \frac{y - xy'}{y^2} &= 1 - y' \\ e^{x/y}(y - xy') &= y^2 - y^2y'\end{aligned}$$

Solve for y' .

$$\begin{aligned}ye^{x/y} - xe^{x/y}y' &= y^2 - y^2y' \\ ye^{x/y} - y^2 &= (xe^{x/y} - y^2)y' \\ y' &= \frac{ye^{x/y} - y^2}{xe^{x/y} - y^2}\end{aligned}$$